

# Phase Transitions in a Probabilistic Cellular Neural Network Model Having Local and Remote Connections

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**Abstract**— Inspired by a neuronal architecture, we show how to produce dynamical behavior in a special kind of probabilistic cellular neural network system. We demonstrate that the spatial and temporal behavior of neural activity undergoes sudden changes if the connection structure and noise component are varied. We characterize quantitatively phase transitions using the activation and cluster size. We indicate the potential role our present results may play in developing the theory of computation using non-convergent neurodynamic principles, called **neuropercolation**.

## I. INTRODUCTION

Methods of multi-component stochastic systems theory are applied in many parts of biology on the molecular, cellular, genetic, and population levels; in computer science; in polymer chemistry; in seismology; in economics, and other sciences. From the probabilistic point of view, such systems are special cases of multidimensional Markov processes [1]. Two main principles are applied. The systems are treated as multi-component and homogeneous, i.e. all components have similar rules of behavior. Systems like these can be used to describe central nervous system functioning [2], [3]. There is an interest in phase transitions, that is, discontinuous changes of the system's behaviors as a result of continuously changing parameters [4], [5]. The purpose of this study is to get a better understanding of phase transitions in the brain and brain functioning. Our studies lay the foundations of a new type of neural network [6] with non-convergent dynamical encoding of input data. Encoding and fast recall of information is based on phase transitions in the dynamical behavior of the neural network. In this work, we study certain spatial-temporal characteristics of phase transitions in a single-layer NN. Future studies will be conducted on extending these results to multi-layer stochastic systems that model brain functions.

## II. MODEL FOR RANDOM CELLULAR AUTOMATON AND PHASE TRANSITIONS WITH REMOTE CONNECTIONS

### A. Inspiration from Dynamics of Neurons and Neuron Populations

There are two main types of neurons in the cerebral cortex [7]. Projection neurons have a dendritic arbor that can grow to a diameter of up to a millimeter. The axon of a projection neuron extends beyond the dendritic arbor and it can extend

nearly half the length of a person's body. The second type of neuron is the interneuron, whose dendrites form a densely branched arbor extending out in all directions to a diameter of about a tenth of a millimeter.

Each neuron keeps the integrity of its membrane over all of its filaments, and its axon tips make contact with other neurons by synapses. Synapses increase or decrease the target activity. Since neurons inhibit or excite through synapses, we can refer to them as excitatory or inhibitory neurons.

Each neuron continually converts incoming pulses to waves, sums them, converts its integrated wave to a pulse train, and transmits that train to all its axonal branches. Microscopic pulse and wave state variables describe the activity of the single neurons that contribute to the population, and mesoscopic state variables describe the collective activities to which the neurons give rise, also in the pulse and wave modes.

### B. Model Description

The model introduced in this paper has only excitatory neurons represented by the vertices in the lattice. Each site has four neighbors, which can be local or remote ones, see Figures 1 and 2. The lattice is folded into torus, which means that the first row/column sites are the down/right neighbors for the last row/column and the last row/column sites are the up/left neighbors for the first row/column. This corresponds to periodic boundary condition. Remote neighbor to the site can be any site in the lattice. If the site has remote neighbors, it loses as many local ones as there are remote ones by random choice. There are a maximum of four remote neighbors per site. The number of remote neighbors is given as the initial condition parameter. Each site can be in one of two states: active or inactive. At each time step the site changes its state according to the transition rules. Activation values at each step are given as:

$$a_{ij}(t+1) = f(\sum_{\Lambda \in n(i,j)} a_{\Lambda}(t)) \quad (1)$$

Here  $a_{ij}(t)$  is the activation value of node at location  $\{i, j\}$  at time  $t$ .  $f$  is a nonlinear function representing the transition rule, which is a majority rule.  $f$  takes the same value as the majority of sites in the neighborhood.  $n(i, j)$  denotes the neighborhood of  $\{i, j\}$ . For example, in the case of local interaction the neighborhood includes 5 elements:  $\Lambda = \{\{i-1, j\}, \{i+1, j\}, \{i, j\}, \{i, j-1\}, \{i, j+1\}\}$ . We use self-connections for convenience, although it is not biologically plausible.

Example of the neighborhood in case of two additional remote neighbors for the node at  $\{i, j\}$  would be:  $\Lambda = \{\{i, j\}, \{i, j-1\}, \{i, j+1\}, \{k, l\}, \{m, n\}\}$ . Local sites  $\{i-1, j\}, \{i+1, j\}$  were cut out by random choice.

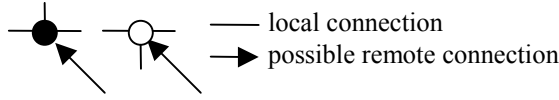


Fig. 1. Examples of two nodes with local and remote connections: black - inactive, empty - active site.

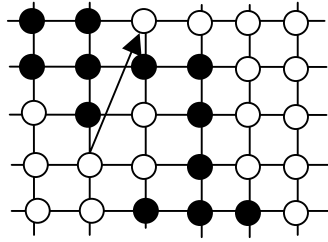


Fig. 2. Schematic view of a small lattice with one remote connection.

### C. Assignment of Remote Neighbors and Transition Rules

Initially remote neighbors are picked randomly and after that their locations remain unchanged. Each site has an exactly predetermined number of remote neighbors. If 10% of the sites have one remote neighbor, then in the 10X10 lattice (100 sites) exactly 10 sites will have exactly one randomly selected remote neighbor (in addition to 4 local). Connections with the remote neighbors are unidirectional, that is a remote neighbor acts as the neighbor to the site, but the site itself is not a neighbor to the remote neighbor. There is a probabilistic component  $\varepsilon < 0.5$ , the initial value parameter.  $j$  denotes a chance of the site following the majority rule:

$$j = \left\{ \begin{array}{ll} 1 - \varepsilon; & \text{the value of majority} \\ \varepsilon; & \text{the value of minority} \end{array} \right\} \quad (2)$$

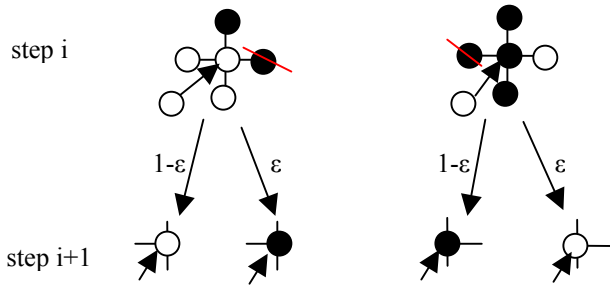


Fig. 3. Schematic view of one example of transition rules.

### D. Statistical Description

We define the parameters used in the paper in the Table I. Graphical representation of the parameters is shown in the Figure 4.

TABLE I

DEFINITION OF THE PARAMETERS	
Notation	Description
$N$	number of sites in a lattice
$\varepsilon$	the probability of the site to change its state
$S_j$	sum of all active sites at time $j$
$d_j$	$S_j/N^2$ : density of active sites at time $j$
$d_{\min}$	min value of $d$ in the experiment
$d_{\max}$	max value of $d$ in the experiment
$\Delta$	$d_{\max} - d_{\min}$
$d_c$	range between $d_{\min} + 0.25\Delta$ and $d_{\min} + 0.75\Delta$
$\mu$	average density for given $\varepsilon$ for large $N$ ( $N \rightarrow \infty$ )
$T$	number of runs
$T_c$	the number of $d$ occurrences in $d_c$
$\gamma_i$	jump event, when average density of previous runs, ( $\mu$ or $1-\mu$ ), becomes $1-\mu$ or $\mu$ for the runs that follow
$\tau_i$	wait time for the jump $i$
$\langle \tau \rangle$	$\sum_i \tau_i / T$ : average wait time for jumps
$\theta_i$	number of steps required to complete a jump from state $i$ to state $i+1$ , (transition)
$\langle \theta \rangle$	$\sum_i \theta_i / T$ : average number of steps for transition
$\varepsilon_c$	critical $\varepsilon$ is where $T_c \gg T/2$

For very large  $N$  jumps are very unlikely to occur due to the probabilistic nature of the model. As  $N$  decreases, the likelihood of jumps is getting larger. In the experiments we try to show basic relationship between the jumps, lattice size, number of remote connections, and probabilistic component  $\varepsilon$  in terms of the density values. We also show the relationship between the critical  $\varepsilon$  and the structure of remote connections.

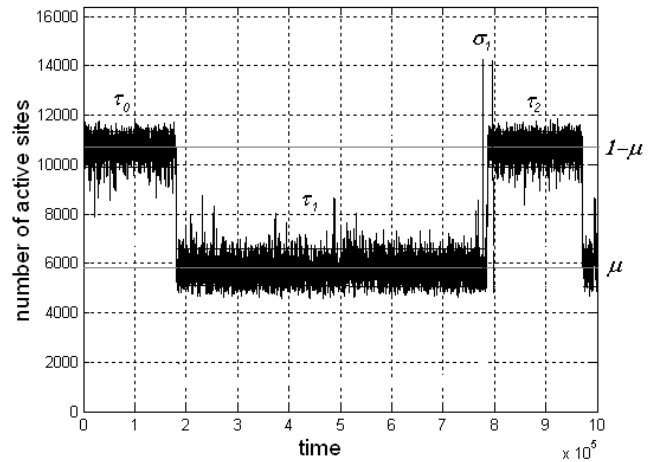


Fig. 4. Graphical indication of the major temporal parameters of phase transitions.

### III. TEMPORAL DYNAMICS OF ACTIVE SITES

#### A. Simulation Results on Densities

The density changes with time and  $\epsilon$ . By plotting density levels in time we observe jumps. For various  $\epsilon$  levels different systems jumps differently.

Different systems in the paper refer to systems with different structures of connections and/or different number of remote connections. E.g. 10X10 lattice with 10% (1) remote neighbors is equivalent to 20X20 lattice with 40 randomly selected remote neighbors, but it is not equivalent to 10X10 lattice with 5% (1) remote neighbors or 5% (2) remote neighbors.

Figures 5 show examples of behavior of different systems. Small squares in the picture show the typical spatial configuration of the active sites for the given  $\epsilon$  level. Examples in Figures 5 are on the 128X128 lattice.

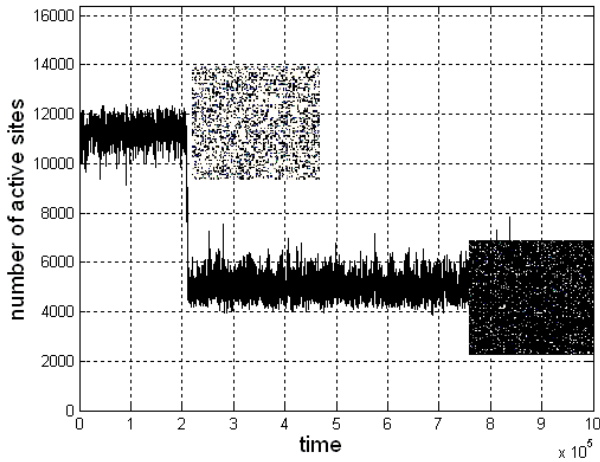


Fig. 5a. Temporal behavior of the active sites.  $\epsilon = 0.145$ . 5% of sites have 1 remote neighbor. The squares represent typical spatial configurations.

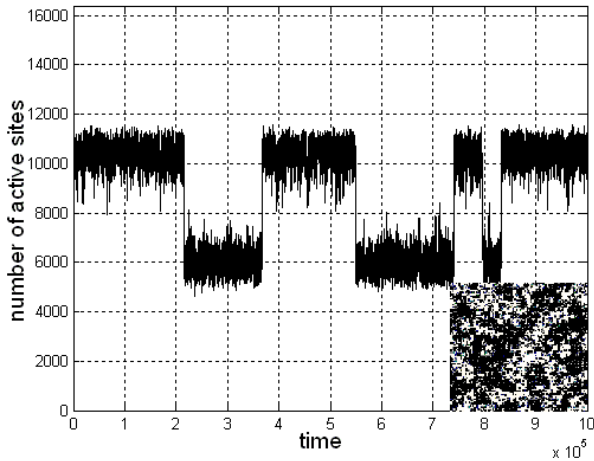


Fig. 5b. Temporal behavior of the active sites.  $\epsilon = 0.17$ . 16% of sites have 1 remote neighbor. The square represents typical spatial configurations.

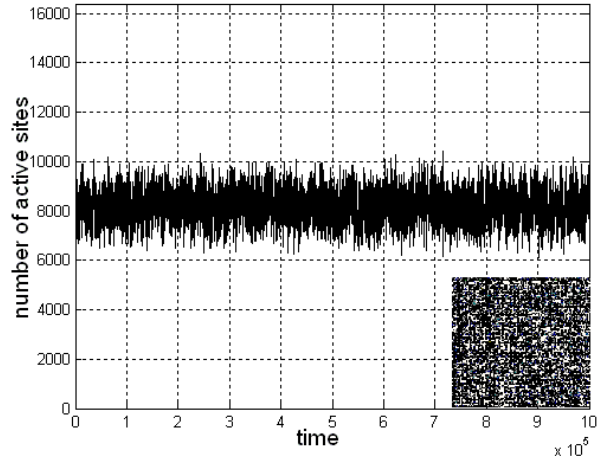


Fig. 5c. Temporal evolution.  $\epsilon = 0.16$ . 10% of sites have 1 remote neighbor. The square represents typical spatial configurations.

We plotted the average density ( $\mu$ ) and the  $\epsilon$  level for different systems. Coordinates were obtained by running the experiment on 128X128 lattices. To obtain  $\mu$  for the given system the experiment were run for 1 million runs. For comparison, we show four characteristic systems in the Figure 6. It can be seen that the addition of the remote connections shifts the critical  $\epsilon$ .

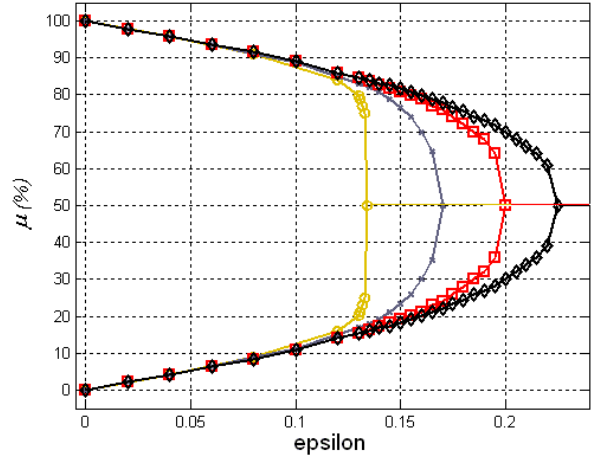


Fig. 6. Average density as function of  $\epsilon$  for the systems with no remote neighbors, 25% of one possible remote neighbor, 100% of one remote neighbor, and 100% of four remote neighbors, (from left to the right).

#### B. Experiments with Critical $\epsilon$

To find the critical  $\epsilon$  we ran experiments for the 1 million iterations for the systems with different combinations of remote connections. Results are shown in the Figure 7, where

the horizontal line shows the probability of the model having different combinations of remote connections and the vertical line shows the critical  $\epsilon$ .

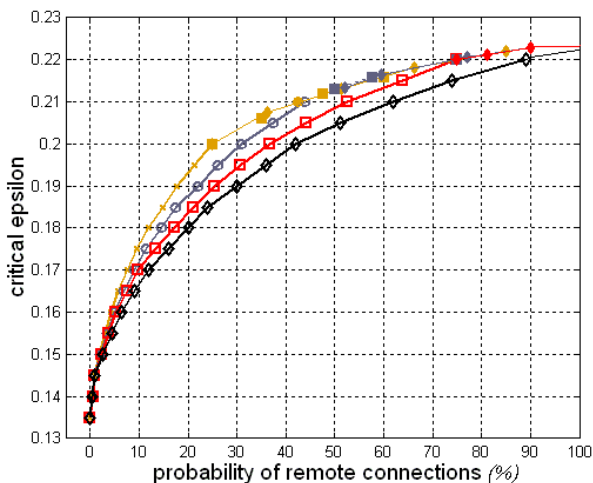


Fig. 7. Critical  $\epsilon$  as the function of the probability of having remote connections.

In the Figure 7, 100 on the horizontal line represents the system where every site has 4 remote connections, which is 4 times  $N$  of remote connections. For example, the value 25 on the horizontal line in the Figure 7 represents all possible combinations of remote connections that add up to 25% of  $N$  times 4 remote connections, e.g.: 100% of 1 remote connection or 25% of 4 remote connections or 50% of 2 remote connections or 50% of 1 remote connections and 25% of 2 remote connections.

Critical  $\epsilon$  does not depend on the lattice size, but it depends on the structure and number of connections among the neurons in the system.

#### IV. QUANTITATIVE CHARACTERISTICS OF AVERAGE WAITING TIME AND RESIDENCE TIME

To obtain  $\langle \tau \rangle$  and  $\langle 0 \rangle$ , the models were run for one billion steps.  $\mu$  values were picked based on the experimental runs; they are lower for the lower  $\epsilon$  level and higher for the higher  $\epsilon$  level. Relatively, for the given  $\epsilon$  and the connection structure,  $\mu$  values are same. Different systems will have different wait time for the jump and different number of steps required for transition for the same level of  $\epsilon$ . Results are shown in Figures 8 and 9. We have the following observation:

- Given the  $\epsilon$ , which is lower than the critical  $\epsilon$ , more jumps occur per number of runs in the lattice with fewer sites, (shorter wait time in the smaller lattice given the fixed  $\epsilon$ ).
- Given the lattice, the higher the  $\epsilon$  up to the critical one the more jumps occur per number of runs, (shorter wait time with higher  $\epsilon$  given the fixed lattice size).
- Given the  $\epsilon$ , which is less than the critical one,

lattice with more sites needs more runs to accomplish the transition.

- Given the lattice size without enough remote connections, for the  $\epsilon$  level below the critical one, the lower the  $\epsilon$  level the higher the number of runs to accomplish the transition.
- Wait time for the jump is shorter within same relative distance from the critical  $\epsilon$ , for the systems with more remote connections.
- With enough remote connections, time needed for transition stays about the same for any  $\epsilon$  level.

With the remote connections the system is “richer”. This means that it jumps more frequently and that it has higher number of different amplitudes within same relative distance from the critical  $\epsilon$  when compared with the system without remote connections.

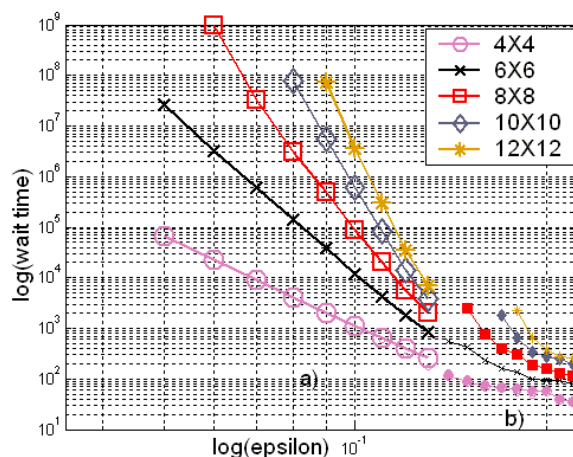


Fig. 8. Average waiting time for jump in systems with no remote connections (a); and with (100%) 4 remote connections (b) vs.  $\epsilon$  in log-log scale.

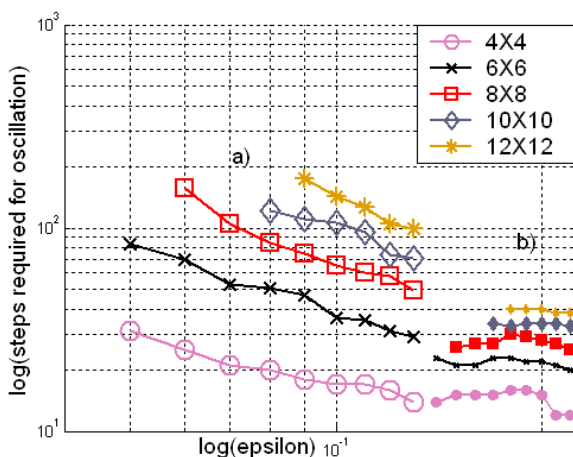


Fig. 9. Number of time steps required to reach transition for given  $\epsilon$  and lattice size for system with no remote connections (a) and 100% (4) remote connections (b) respectively in log-log scale.

## V. CONCLUDING REMARKS AND FUTURE PLANS

In this paper we have studied phase transitions in a neural network model with randomly initiated possible remote connections, which describe long-range axons in biological neural systems. The probabilistic feature of the network is described with the value of  $\epsilon$ . Different systems will exhibit different jumps for the same level of  $\epsilon$ , as it is seen in Fig. 7.

Our observations indicate that adding remote neighbors causes the increase of critical  $\epsilon$  level. Moreover, there is lower and upper bound for the critical  $\epsilon$ , when the lattice has no remote neighbors and when lattice is fully connected respectively. It is possible to simulate critical  $\epsilon$  between lower (0.13428) and upper bound (0.233) to any desired precision by arbitrary lattice size and connection architecture. Remote connections change dynamics of the system. There are more jumps within same relative distance from the critical  $\epsilon$  level in the lattice with more remote connections.

The model described above oscillates due to the probabilistic component  $\epsilon$ . The amplitudes and the frequencies of the jumps will depend on the connection structure of the model. Model with the set up described in the paper creates the jumps for the network as a whole. Without the probabilistic component, the network needs another two layers to cause the jumps, one excitatory layer and one inhibitory layer. These new jumps we call oscillations. The first layer has the sites that excite the sites connected to it and the other inhibitory layer, inhibits the sites connected to it. Behavior of the oscillations depends this time not on the probabilistic component, but on the level of excitation of the sites in the layer. It depends on the input stimuli. The construction of these multi-layer units is in progress.

The multi-layer system has the potential of conducting computations. The operation of such a computational system is based on the oscillations. The oscillating system is waiting for the input that excites the network of neurons. The neurons interact according to the connection strengths. If the system starts to oscillate differently with a given magnitude exceeding a threshold value, we conclude that it recognizes the input. Alternatively, the system may stay in the current oscillatory state after receiving the stimulus. This means that the input is new, and a learning process can be initiated.

When developing an operational neural network model based on the present work, the connections of simultaneously stimulated neurons should be adjusted gradually (Hebbian learning). This makes system react selectively depending on the actual input pattern. Changed oscillations after the given input would mean that the network have successfully learned the given pattern. Adaptation of the described network connections is an important task of future study.

## ACKNOWLEDGMENTS

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