

# Self-Organized Development of Behaviors in Spatio-Temporal Dynamical Systems

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**Abstract - Nonlinear distributed modeling is applied to generate conditions for the emergence of intentional behaviors afforded by the environment. The models are based on a nondeterministic dynamical approach to self-organized formation of categories using chaotic principles. Continuous and discrete models of the spatio-temporal dynamics are shown to exhibit phase transitions manifested in the form of intermittent spatio-temporal structures, which are studied in simulated environments.**

## I. INTRODUCTION

Intelligent behavior is characterized by flexible and creative pursuit of endogenously defined goals by the organism. It has emerged in humans through the stages of evolution that are manifested in brains and behaviors of other animals. Intentionality is a key concept by which to link brain dynamics to goal-oriented behavior. The basic form of intentional behavior is an act of observation through space and time, by which information is sought for future action (Freeman, 2000). Sequences of such acts constitute the key desired property of free-roving, semi-autonomous vehicles capable of exploring remote environments that are inhospitable to humans. Intentionality consists of the neurodynamics, but which images are created for future states and goals, of command sequences by which to act to pursuit of goals, of predicted changes in sensory input resulting from intended actions by which to evaluate performance, and modification of the device by itself for learning from the results of actions.

These principles are well-known among psychologists, cognitive scientists, and philosophers. The new aspect of our approach is the development of a novel tool of mesoscopic/intermediate-range brain dynamics using nondeterministic chaos theory to understand and simulate the construction of meaningful patterns of neural activity of sensory perception (Freeman and Kozma, 2000). We build on neurologically inspired, bottom-up approaches of embodied category formation (Thelen and Smith, 1994; Kelso, 1995; Clark, 1997; Almassy, Edelman, and Sporns, 1998; Freeman, 1999; and others). We show that mathematical theory of random graphs and percolation processes (Bollobas, 1985) can pro-

vide a very powerful support to understand and interpret spatio-temporal aspects of neurodynamics. Phase transitions in random cellular automata models are described in the second part of the paper as an example of the nondeterministic spatio-temporal dynamics.

## II. BACKGROUND ON NEURO-PERCOLATIONS

What distinguishes brain chaos from other kinds is the filamentous texture of neural tissue called neuropil, which is unlike any other substance in the known universe (Freeman 1995, Freeman, 1999). Neural populations stem ontogenetically in embryos from aggregates of neurons that grow axons and dendrites and form synaptic connections of steadily increasing density. At some threshold the density allows neurons to transmit more pulses than they receive, so that an aggregate undergoes a state transition from a zero point attractor to a non-zero point attractor, thereby becoming a population. Such a property has been described mathematically in random graphs, where the connectivity density is an order parameter that can instantiate state transitions (Erdos and Renyi, 1960, Bollobas, 1985). In this context, state transitions in neuronal populations will be interpreted as a percolation phenomenon progressing in the neuropil medium.

The dendritic currents of single neurons that govern pulse frequencies sum their potential fields in passing across the extracellular resistance, giving rise to extraneuronal potential differences manifested in the EEG, which correspond to the local mean fields of pulse densities in neighborhoods of neurons contributing to the local field potentials. In early stages of development these fields appear as direct current "d.c." fields with erratic fluctuations in the so-called "delta" range, below 1 Hz. The neurons are excitatory, and their mutual excitation provides the sustained aperiodic activity that neurons require to stay alive and grow. Unlike transistors, neurons have a short shelf life if they are isolated and left inactive. The activity of an excitatory population is self-stabilized by a non-zero point attractor (Freeman 1975), giving rise to a field of nearly white noise, up to a frequency limit determined by the duration of the action potentials. At some later stage, typically in humans after birth, cortical inhibitory neurons develop or transform

from excitatory neurons, which contribute negative feedback, leading to the appearance of oscillations in the gamma spectrum of the EEG. The mutual excitation persists, and, in fact, is essential for the maintenance of the near-linear range of cortical oscillations through a depolarizing bias (Freeman, 1999).

This self-sustaining, randomized, steady state background activity is the source from which ordered states of macroscopic neural activity emerge, like the patterns of waves at the surfaces of deep bodies of water. Neural tissues, however, are not passive media, through which effects propagate like waves in water (Freeman and Kozma 2000). The brain medium has an intimate relationship with the dynamics through a generally weak, subthreshold interaction of neurons. The synaptic interactions of neurons provide weak constraints on the participants, and the resulting covariance appears in the form of spatiotemporal pattern. In the framework of this research we use mathematical tools of percolation theory to explore the information processing mechanisms in brains as the randomized activity of myriads of neurons that provides the basis for self-organization and higher brain functions.

Information processing in the proposed chaotic dynamical memories is closely related to percolation phenomena (Bollobas and Stacey, 1997; Kauffman, 1990). For about forty years now, percolation theory has been an active area of research at the interface of probability theory, combinatorics and physics (Grimmett 1999). Interest in various aspects of standard percolation remains high, including estimates of critical probabilities (Balister, Bollobas and Stacey 1993; Balister, Bollobas and Stacey, 2000). In the last decade and a half, more and more variants of the standard percolation models have been studied. In particular, there has been much work on the family of processes known as bootstrap percolation (Aizeman and Lebowitz, 1988; Duarte, 1989; Adler, 1991; Schonman, 1992; Cerf and Cirillo, 1999, and many other papers). Computer experiments performed by physicists have suggested interesting non-trivial large-scale behavior, and many deep mathematical results have been proved about a number of models.

Percolation theory deals with large scale properties of certain types of random graphs, often built from lattices such as  $Z^n$ . In the archetypal percolation problem the vertices (sites) are the points of a lattice such as  $Z^n$  with edges or bonds joining neighboring sites. In site percolation, sites are open independently with probability  $p$  and one wishes to answer questions about the size of the connected components formed by these open sites. In particular, do infinite connected clusters of open sites exist? Similar questions can also be asked about bond percolation, where the bonds are chosen to be open with a certain probability. There are many variants of these problems. For example, in oriented percolation one asks for infinite paths of connected open sites that travel at each step only in certain directions. Many percolation problems exhibit

phase transitions, where for  $p$  less than some critical probability  $p_{crit}$  only finite clusters exist, and for  $p > p_{crit}$  infinite clusters almost surely exist. Random cellular automata generalize deterministic cellular automata such as Conway's game of life. Given the complexity of deterministic automata it should not come as a surprise that random automata display extremely complex behavior that will be proved to be crucial in understanding brain functions.

### III. PHASE TRANSITIONS IN PERCOLATION MODELS

On a finite grid or torus, we can compare random automata with the corresponding *mean field* model. In the mean field model instead of taking a set of specified neighbors, we take a subset of the elements of the grid at random (with replacement). It is clear that the mean field model does not depend on the topology of the grid, and the only information of relevance to the site activity vector  $X_t$  is given by its cardinality  $|X_t|$ . We define  $x_t$  to be  $|X_t|/N$  where  $N$  is the size of the finite grid or torus. Thus  $x_t \in [0, 1]$  gives the density of points in  $X_t$ .

Our real interest lies in random cellular automata on  $Z^2$  rather than the mean field models (Balister et al, 2001). So one question is how well do the mean field models approximate the corresponding random automata. Unfortunately, the answer is 'not very well in general'. Since the mean field model is equivalent to that of a fully isotropic model, the best we could hope for is that it approximates fully isotropic models. However, even fully isotropic models can exhibit behavior that is richer than that of the mean field models.

For example, take the symmetric fully isotropic model with  $p_0 = \frac{15}{16}$ ,  $p_1 = p_2 = \frac{1}{256}$ . Fig. 1 shows a typical state of this model on the  $256 \times 256$  torus. We can show that this map has two stable fixed points and an unstable fixed point at  $x = 0.5$ . Thus the mean field model will have two phases, one of low density and the other of high density. The random automata does show distinct phases, but they all have densities of 0.5. Indeed there seem to be at least 8 different phases, none of which are isotropic. This is an example of spontaneous symmetry breaking.

The model is symmetric under interchange of coordinates, but some phases consist of horizontal lines and some phases consist of vertical lines. The lines are of width two, so for each orientation there is a choice of alignment mod 4 in the vertical or horizontal direction, giving 8 phases in total (Kozma, Balister, Bollobas, Freeman, 2001; Balister, Bollobas, Kozma, 2001).

For another example, consider the symmetric fully isotropic model  $p_0 = p_1 = p_2 = p$ . For  $p < 0.5$  this is an 'Ising'-like model as we shall see later. For the mean free model

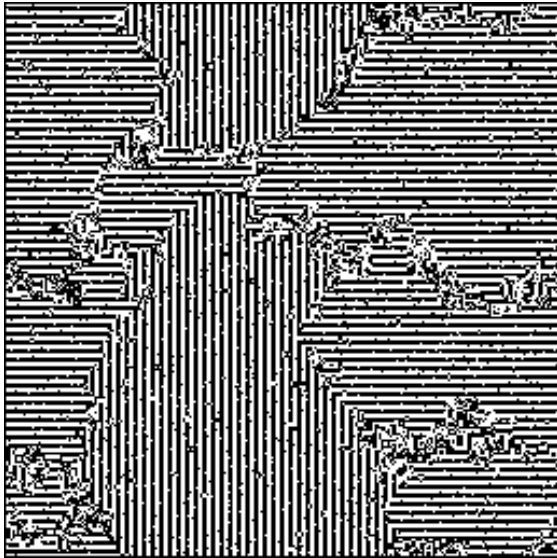


Fig. 1. Fig. 1: Symmetric isotropic model. Note the presence of non-isotropic phases

this has one fixed point at  $x = 0.5$  for  $p \in [\frac{7}{30}, \frac{1}{2}]$ , but for  $p < \frac{7}{30} = 3D.2333$  the fixed point  $x = 0.5$  is unstable and there are two other fixed points which are stable. The random automata behaves qualitatively similarly, except that the critical probability is significantly lower at about  $p_c \approx 0.13$ . For  $p_c < p < 0.5$  it appears that the stationary density distribution of  $x_t$  for a sufficiently large but finite lattice is unimodal with peak at  $p = 3D0.5$ . For  $p < p_c$  the distribution becomes bimodal, as one would expect from the mean field model, and there are two phases in the infinite lattice, one with high density and one with low density. The continuous time model behaves similarly, except that the critical probability is even lower, at about  $p_c \approx 0.10$ .

Looking at Fig.2 it is at first sight unclear as to whether there really is a phase transition, since even for  $p > 0.14$  there appear to be regions of high and low density. We give a heuristic reason for believing that there is indeed a phase transition in this model.

#### IV. Conclusions

We propose a novel approach to model natural phenomena, in particular, brain processes. We suggest using statistical processes in the models themselves, replacing stochastic differential equations by random models of elementary interactions between basic components. This way, we can compute dynamical oscillations in the modelled system. In the framework of this research, we use mathematical tools of random graphs and percolation theory to explore information process-

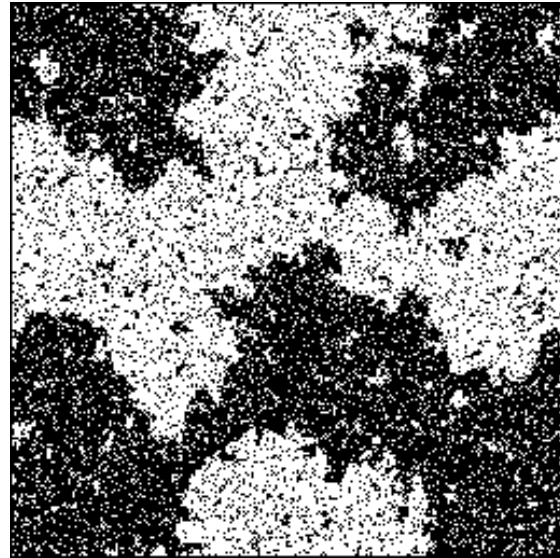


Fig. 2. (a)  $p = \frac{4}{32}$

ing mechanisms in brains. The randomized activity of myriads of neurons provides the basis of formation of neural populations which, in turn, leads to self-organization and the emergence of goal-oriented brain functions.

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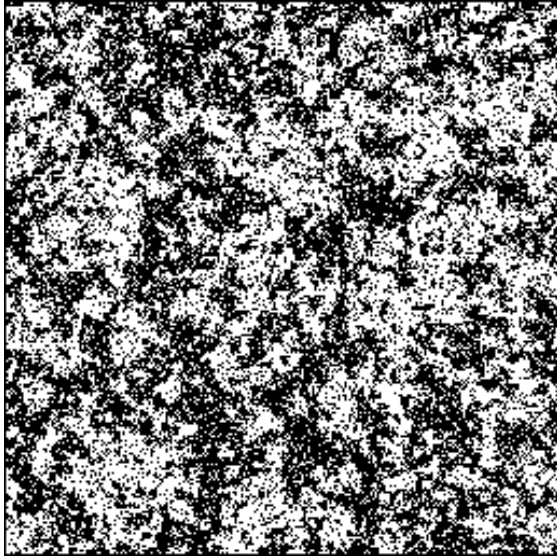


Fig. 3. Fig. 2: Symmetric isotropic model near the phase transition; see case (b) above:  $p = \frac{5}{32}$

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