Activation Clustering in Neural and Social Networks

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Questions related to the evolution of the structure of networks have received recently a lot of attention in the literature. But what is the state of the network given its structure? For example, there is the question of how the structures of neural networks make them behave? Or, in the case of a network of humans, the question could be related to the states of humans in general, given the structure of the social network. The models based on stochastic processes developed in this article, do not attempt to capture the fine details of social or neural dynamics. Rather they aim to describe the general relationship between the variables describing the network and the aggregate behavior of the network. A number of nontrivial results are obtained using computer simulations.

Key Words: social networks; neural networks; state transition; RCA; percolation

1. PHENOMENOLOGY OF NEURAL AND SOCIAL NETWORKS

1.1. Introduction

Neural and social networks have several common features. In both networks, the individual entities mutually influence each other as participants in a group. Although a social network is made up of humans, a neural network is made up of neurons. Humans interact either with long-reaching telecommunication devices or with their biologically given communication apparatus, whereas neurons grow dendrites and axons to receive and emit their messages. Dendrites and axons grow and decay continuously after birth, whereas people are making connections in the form of their relationships during their lives. Individuals are different concerning the ways they communicate. We distinguish two main types of neurons. Local neurons or interneurons are one type, and projection neurons are the other type. The interneuron is like a local business or locally interacting individual, having connections within a local neighborhood. The projection neuron is like a globally important person, having long-range connections across far reaching neighborhoods. The dendritic trees in neural networks continuously change the contacts with axons. People continuously develop their relationships by connecting or disconnecting with others. The competition for con-

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nection space is intense, and success in finding and maintaining a connection depends on the activation of relationships. If the relationships are inactive, the connections decay and the neuronal or social synapses disappear. This study describes the short-term activation clustering in networks, with a help of Tobochnik's insights into computer simulation in [1]. We assume that neural and social network structures change slowly in comparison with the speed at which network components communicate.

1.2. General Network Model

There are different levels of abstractions for neuronal and social network models. One abstraction uses a network of interconnected sites. In the general sense, a network is a collection of objects connected to each other in some fashion. In this case, the network objects are sites or nodes, representing cells, persons, or firms. Connections in network models represent influences of sites in the network. A network or system evolves and changes in time, driven by the activities or decisions of its components. When reduced to basic variables, the system is described by sites, connections among sites, and the rules the sites obey in time. A site, which expresses its level of activations through its states, is directly connected to its neighbors through a single connection. A site can be influenced by, or can influence, a different number of its neighbors. If there are two sites mutually influencing each other, the connection between them is two-way. If a site either only receives or only sends the influence to its neighbor, the connection is one-way. The direction of a one-way connection depends on which site is influencing and which one is being influenced.

1.3. Network Growth Models

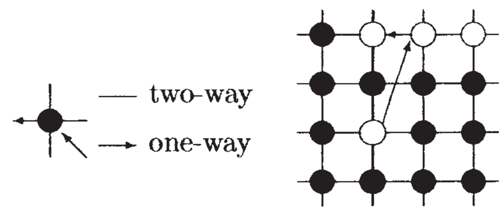
Network structure grows and changes in time. In the many existing network growth models, the continual addition or removal of both vertices and edges and preferential attachment or removal of those were considered to analyze the evolving network structure. For example, see [5–8]. Fundamental results on scale-free networks are given by Albert and Barabasi [2] and by Bollobas [3, 4]. Even though the interest of this article is on the evolving activation states of the network once the network is built, it is still important to understand the network growth model, so the proper principles can be applied for building the model. Usual growth models mentioned are inappropriate for social and neural networks for the reasons mentioned in [9]. First, the time scale on which people make and break social connections is not the same. Second, the degree distribution of many acquaintance networks or neural networks does not follow a power-law distribution. Third, for two of one's friends to be friends themselves is common in social and neural networks, whereas in many growth models such clustering is not common [10, 11]. Most importantly for this article, the social and neural network sites have one-way influences. One-way influence means that the influenced site does not directly influence the site that targets it. For example, a strange person on television influences the opinion of the watcher, without being directly influenced by the watcher himself. A neuron influences a remote neuron by an axon, without being directly influenced by the remote one.

In this article the goal is to measure the network states and build a model based on lattices on two-dimensional grid. It is reasonable to represent neural and social networks with a lattice, in which sites represent neurons or people and the links represent the connections or influences among the sites. Because societies and neural populations have individuals that influence the others directly, while not being influenced by those others directly back, the one-way connections should be added to the regular lattice. States of the network components can be simplified to the extreme, as being active or inactive. The modified lattice should not structurally change in time, because the functional relationships between the variables that describe the structure of the system and the system global activation behavior are of interest.

1.4. Rules of Behavior

In the social network one's opinion depends on what other people think. Humans have to take the preferences of others into account when devising the plan of their actions. In the real world, many problems are either too complicated or too uncertain for an individual to evaluate. Sometimes there is a lack of adequate information, so there is a need for help in decision making. In other cases, there is too much information and individuals lack the ability to process it effectively. In the 1950s, Herbert Simon proposed that people try to behave rationally, but their capacity to do so is bounded by cognitive constrains and limited access to information [12]. People listen to others routinely and it works well. Consequently, people have a reflexive tendency to accommodate to the actions of others. In the modeling of social network states, there is a need to come up with a rule that describes the tendency of people to follow the majority influences they receive. It is simply called majority rule. It is obvious that the majority rule can nicely simplify the model of neuron activation. Every neuron simply increases its activity if the majority of its neighbors do so, and every neuron decreases the activity if the neighboring neurons decrease their activity. In very complex systems, like biological and social, there could be a multitude of influences on an individual, so it becomes hard to figure out who is exactly being influenced by whom. To overcome this difficulty, some randomness in the decision making rule is introduced. If the most important influences are from the neighbors, a rule says that a site most likely responds to the influences of its neighbors, yet there is a chance that a site might not follow the neighbors' influences. This is modeled in random cellular automata on lattices [13].

FIGURE 1



An example of inactive site with possible connections (left), and a lattice with a remote connection.

1.5 Questions to Be Studied

Models of social and neural networks need to answer the questions about social and neural structure, synchronization, and subcomponent clustering. For example, given the structure and the rules in the time dimension, what is the asymptotic behavior of the system in terms of its activation and its subcomponent clustering? Under which conditions does population of asynchronous individual components become synchronized? Or, when do the activity patterns of subcomponents become alike, so the system exhibits globally synchronized behavior? What is the short-term dynamics of the system, where "short-term" describes the system that does not change its structure due to learning? Important questions are related to state transitions or sudden state changes in the system. In [14] and [15], the activity of a brain is described with itinerant trajectory over its landscape of attractors. There is a succession of momentary pauses in the basin of attractors or states to which the brain travels. The states jump between each other by state transitions in chaotic itinerancy. Each itinerant step is a global state transition. State transitions occur in social networks too. They are manifested in the bubbles of financial markets, fashion fads, revolutions, and so on. Studying relationship between the system's stochastic components with homogeneous rules of behavior and the emergent behavior they produce improves our understanding of the state transitions in neural and social systems. Methods that encompass the motivations from biological neurons, percolation theory, random graph theory, and statistical physics, help to explain what occurs in the moments of change and why complex phenomena happen.

2. MODEL DESCRIPTION

2.1. Description

Let a model be a lattice with excitatory sites, which can be active or inactive (Figure 1). Each site at *i, j* has four local connections: up, down, left, and right. A site can have remote connections, which are any nonlocal connections. Each site with remote connections loses, by random choice, as many local connections as there are remote ones. The collection of all the sites with connections to the site at *i, j* is called neighborhood $n(i, j)$.The lattice with local connections only is folded into torus, so the first row/column sites are the down/right connections for the last row/column, and the last row/column sites are the up/left connections for the first row/column. The locations of remote connections are fixed and chosen randomly at the initiation. Remote connections are unidirectional. In discrete time, a site's state a_{ij} at location *i, j* is defined by the majority rule, which tells a site to have the activation that most of its neighbors have (see Equations 1 and 2). All sites simultaneously check their neighborhoods at time *t*, and set the new activations at time $t + 1$. Let $\epsilon < 0.5$, be the initial value parameter that determines the probability of a site to change its activation. ϵ in the model represents a tendency of an individual not to be influenced by directly connected neighbors. This is realistic for social and neural networks, because the decisions in those systems are too complex to be considered as completely deterministic. The rule described obeys majority rule that is applicable in most models of social and neural networks. Because there is no concern of structural change in time, every site in a two-dimensional environment has a unique non-changing neighborhood.

$$a_{ij}(t+1) = \begin{cases} \textit{minority at t w/p } \epsilon \\ \textit{majority at t w/p } 1-\epsilon \end{cases} \tag{1}$$

or

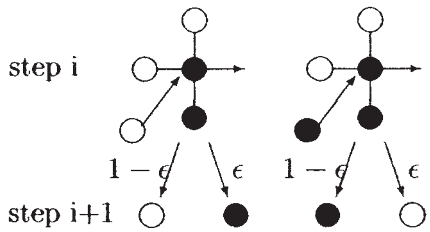
$$a_{ij}(t+1) = \begin{cases} \begin{matrix} 0 \ \ w/p \ 1-\epsilon \\ 1 \ \ w/p \ \epsilon \end{matrix} & \textit{if } \Sigma_{n(i,j)} a_n(t) < \dfrac{|n(i,j)|}{2} \\ \begin{matrix} 0 \ \ w/p \ \epsilon \\ 1 \ \ w/p \ 1-\epsilon \end{matrix} & \textit{if } \Sigma_{n(i,j)} a_n(t) > \dfrac{|n(i,j)|}{2}. \end{cases} \tag{2}$$

$n(i, j)$ denotes the neighborhood of the site at location *i, j*. $|n(i, j)|$ denotes the number of neighbors of a site at location *i,j*. $|n(i, j)| = 5$, for self-connection is assumed. *w/p* means *with probability*. If $\Sigma_{n(I,j)} a_{ij}(t) < |n(i, j)|/2$, the majority is 0 and minority is 1, else the majority is 1 and minority is 0 (Figure 2).

2.2. Connection Structure Description

Connection structure can be described with the number of two- and one-way connections a site has. The more one-way connections, the less structured the network is. A lattice without one-way or remote connections represents perfectly structured network or system (see Figure 3).

FIGURE 2



Examples of transition rules from step *i* to step $i + 1$ for two arbitrary cases. Majority is active on left. Majority is inactive on right.

Let us consider the following four examples: systems with no remote neighbors (*local*), systems with 25% sites having a remote connection [25%(1)], systems with 100% sites having a remote connection [100%(1)], and systems with 100% sites having four remote connections [100%(4)]. In general, different systems refer to configurations with different connection structures; e.g., a 10×10 lattice with 10% of sites having one remote neighbor will have similar behavior as a 20×20 lattice with 40 sites having one remote neighbor, but it is different from the 10×10 lattice with 5% of sites having two remote neighbors.

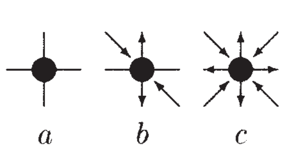
2.3. Activation Density

The number of all active sites at time *j* in the network divided by the total number of sites in the network is activation density at time *j*. Figure 1 shows an example of schematic view of 25% activation density of a lattice folded into torus with a remote connection, where each site has a neighborhood of 5. The activation density of the system varies as a function of ϵ, and generally it is a smooth function. Yet there is a point, called ϵ_c, when the density as the function of ϵ varies drastically! In many physical systems, e.g., water at boiling or freezing point, similar drastic change is expected. In many models, the forces acting between the individual sites vary continuously as ϵ varies. Why is there a sudden change of system state at certain ϵ? This article is devoted to the attempt to understand this behavior.

2.4. Cluster Description

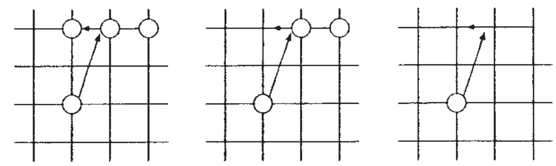
Different from many network models, clustering is defined through the sites' activation, rather then the sites' connections.

FIGURE 3



Examples of neighborhoods with different connections; *a* represents a perfectly structured neighborhood.

FIGURE 4



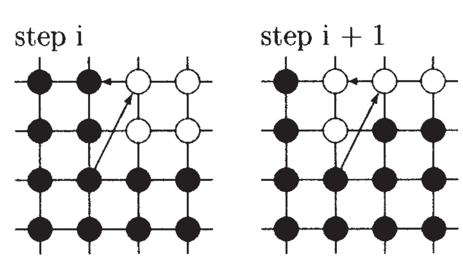
A site in the first row and second column is in the cluster of size 4 (left). A site in the first row and third column, and a site right to it, belongs to a cluster of size 3 (middle). A site that is a remote neighbor makes a cluster of size 1 (right). The average cluster size is 11/4.

This is more useful for the study of network or system states. It is said that a site belongs to a cluster of size *c*, if it has a path via the adjacent connections to the other $c - 1$ sites with the same activation. A minimum cluster has size 1. The other extreme is when all sites in the lattice have the same value. Figure 1 shows an example of schematic view of four clusters. This figure is broken into the parts shown in Figure 4. A site in the first row and second column is in the cluster of size 4, because there is a path through the cluster, so it can gain knowledge about activation values of four sites of same kind. A site in the first row and third column belongs to a cluster of size 3. A site in the first row and fourth column also belongs to a cluster of size 3. A site that is a remote neighbor to the site in the first row and third column does not see the site it remotely influences and cannot gain the knowledge, via the path, about other site's activation value of same kind, so it makes a cluster of size 1. Because there are four clusters of sizes 4, 3, 3, and 1, the average cluster size is 11/4.

2.5. Cluster Overlapping

Cluster overlap represents the percentage of the sites in the network that have the same activation value in the two consecutive steps, t and $t + 1$, normalized by activation density at step $t + 1$ (Figure 5). If the sites are mostly inactive, clusters are made of active sites. If the sites are

FIGURE 5



An example of an overlap of size 50%. Fifty percent of active sites are active at the same locations. Overlap is a measure of how static once formed activation clusters are.

TABLE 1

Definitions of Variables to Describe System's Dynamics

Notation	Description
N_{total}	Total number of sites in a network
l	Lattice size expressed as $\sqrt{N_{total}}$
$N_a(t)$	Number of active sites at time t
$d(t)$	$N_a(t)/N_{total}$ = activation density at time t
$\langle d \rangle$	Average activation density of experiment
$\lvert d(t)_{0.5} \rvert$	$\lvert d(t) - 0.5 \rvert$
$c(t)$	Average cluster size at time t
$c_4(t)$	Proportion of clusters of sizes 4 or greater at t
$\langle c \rangle$	Average cluster size of experiment
$\langle o \rangle$	Average overlap of experiment

mostly active, clusters are made of inactive sites. Overlap is an average measure of the strength of formed clusters to stay intact. The higher the overlaps the less dramatic the change of clusters in the system. In the social network, overlap can be viewed as the measure of strength once created states or opinions of individuals within the group. In the neural networks, overlap is just a measure of the strength of a once formed activation cluster to stay active.

2.6. Statistical Description

Parameters used to describe the clustering are defined in Table 1.

For systems with equivalent structure (equal % of remote connections, but possibly different lattice sizes), the kurtosis values of density as a function of ϵ, differ. There is a point, ϵ_c, where the kurtosis value is the same for all equivalent systems. The idea is explained in more detail in [16] and [17]. The ϵ_c values of many systems are presented in [18]. Table 2 presents ϵ_c values of 4 arbitrary subsystems. ϵ_c is the point of state change. In the social network, ϵ_c is the point at which societies turn to another state of activation or another state of minds. ϵ_c is the place at which some products become fashionable, the place at which financial bubbles burst, or the place where revolutions occur. In the neural system, according to Freeman, Kaneko, and Tsuda [14, 15], ϵ_c is the moment at which the brain states or thoughts jump from one to another.

3. SPATIAL DYNAMICS

3.1 Cluster Sizes of Arbitrary Systems as a Function of Activation Density

To understand clustering, the cluster sizes are measured when different systems have approximately the same average density. When the average density is the same, the measurements show the comparable differences in the average cluster size, the average overlaps, and the cluster distributions of different systems. Typical clustering distributions of the arbitrary systems are shown in Figure 6. Vertical lines in the Figure 6 shows the ratio of the total number of occurrences of a certain cluster size and the total number of clusters that occurred in the experiment. Horizontal lines show the cluster size normalized with the largest possible cluster.

The average cluster size in less structured networks is bigger for the same activation level, when compared to more structured networks (Figure 6). Adding remote connections to the local systems breaks apart the clusters, which spread through the local connection. Remote connections can break the clusters, but can also create possible new ones (e.g., in Figure 1). Importantly, the average cluster size is bigger in the systems with relatively more one-way remote connections.

3.2. Supportive and Unsupportive Clusters

Cluster parts can be supportive or unsupportive. In the supportive cluster parts, all the sites in the cluster have the same activation value as the majority of their neighbors. If a site in the cluster has an activation value different form its neighbors' majority, the site is in the unsupportive part of the cluster. An example of a cluster whose sites support each other to stay together in the cluster, and a cluster whose sites do not support each other as strongly is in Figure 7.

3.3. Why Do Remote Connections Make an Average Cluster Bigger?

The connections are more distributed among the sites in the system when sites have more one-way influences. Figure 3 explains structural differences of neighborhoods, but also shows that with the addition of remote connections, more sites influence each other directly, even though the total number of connections in the system does not change. Every site has the neighborhood of five. With more remote connections, there is less of a connection overlapping, so there is the same number of connections in total, but they directly connect more sites. In the extreme case, *c* in Figure 3, when there are four remote connections per site, the site is one connection away from eight other sites. In the other extreme case, when the system is described by local model, case *a* in Figure 3, the site is one connection away from four sites. Simulation results,

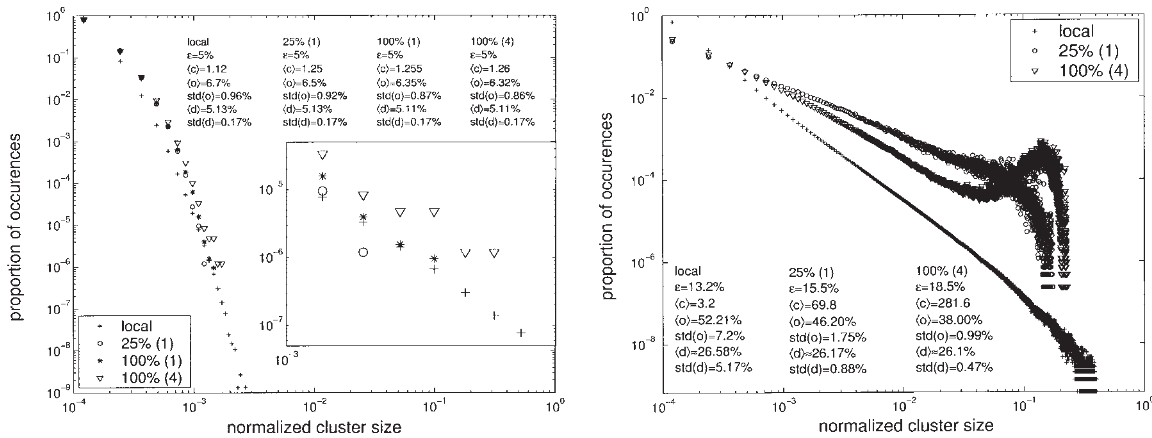
TABLE 2

Arbitrary Subsystems

Local	25% (1)	100% (1)	100% (4)
0.1342	0.1702	0.2032	0.2227

The details are in [18].

FIGURE 6



Average clusters and overlaps for different systems in a lattice size 128x128. Left: $\langle d \rangle \approx 0.05$. Right: $\langle d \rangle \approx 0.26$. As there are more remote connections the links among sites get more distributed. There is less redundancy of connection overlapping. Consequently, $\langle c \rangle$ of the networks with more remote connections is larger, but $\langle o \rangle$ is smaller.

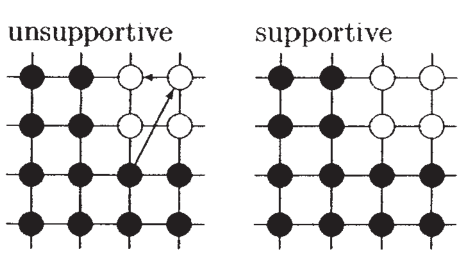
measuring $\langle c \rangle$ as a function of number of remote connections of four typical systems, are in Table 3. To make the arbitrary systems comparable, $\langle c \rangle$ as a function of ϵ is measured keeping $\langle |d|_{0.5} \rangle$ and $\langle d \rangle$ at the same level.

4. MEASUREMENTS OF CLUSTERS AND OVERLAPS

4.1. Average Cluster Size

Both the average activation density and the average cluster size vary continuously as the function of ϵ up to a unique point, ϵ_c, at which the average density and the average cluster size exhibit power law behavior. Figure 8 shows an example of $\langle c \rangle$ and $\langle d \rangle$ in lattice size 128×128. If all the variables describing the system are the same, but its size,

FIGURE 7



An example of an unsupportive (left) and a supportive cluster (right). The majority rule helps the cluster on the right retain its current activation. Remote connections break the majority support for the cluster on the left. In the unsupportive part of the figure, a cluster most likely breaks apart in a series of steps if ϵ is small. For small ϵ, a supportive cluster most likely stays together.

the kurtosis of the system's activation is greater below ϵ_c and lower above ϵ_c in the smaller systems. As the ϵ below ϵ_c increases, the difference between the kurtosis of the smaller system and the kurtosis of the larger system becomes smaller. At ϵ_c the kurtosis is same for any system size. For ϵ greater then ϵ_c, the kurtosis of the smaller system is smaller than the kurtosis of the larger system.

Measure of kurtosis for calculating ϵ_c was used by Binder in [16], also. Binder defines

$$U(l, \epsilon) = \frac{\langle (d - 0.5)^4 \rangle}{\langle (d - 0.5)^2 \rangle^2} \tag{3}$$

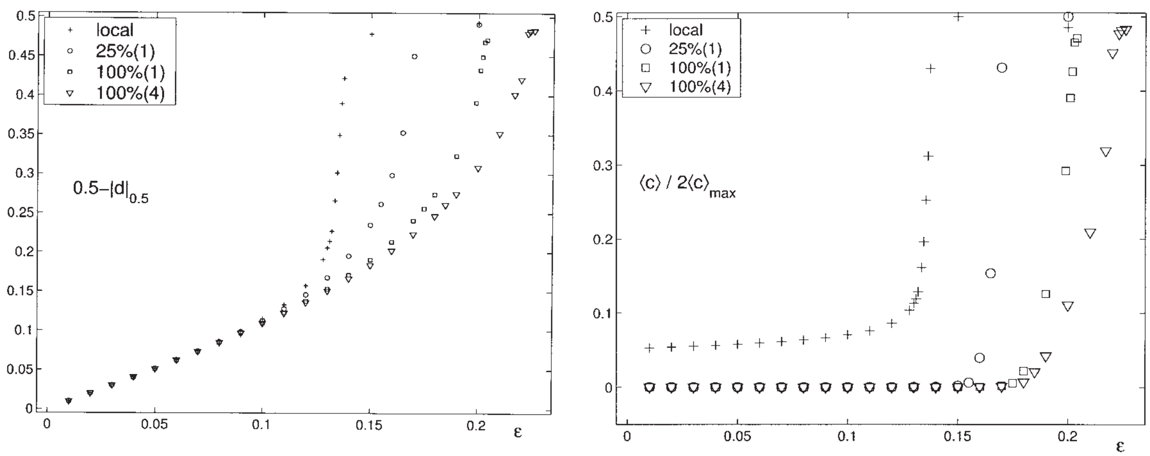
where d in the Equation 3 can be substituted with c to measure the kurtosis with the average cluster size per a step to describe power law behavior with critical exponents ν, β, and γ [19]. A good example by Makowiec is in [17]. Using Equations 4–7 to obtain linear interpolations

TABLE 3

$\langle c \rangle$ of Four Arbitrary Systems in Lattice Size 128×128

	$\lvert d \rvert \approx 5\%$	$\lvert d \rvert \approx 26\%$	$\langle \lvert d \rvert_{0.5} \rangle \approx 11\%$
Local	1.12	3.20	6.02
25% (1)	1.25	69.8	2868.4
100% (1)	1.255	72.4	3779.8
100% (4)	1.26	281.6	4324.1

FIGURE 8



$\langle 0.5 - |d|_{0.5}\rangle$ and normalized $\langle c\rangle$ as a function of ϵ for system without remote connections. $\langle c\rangle$ is normalized, so it is comparable with $\langle 0.5 - |d|_{0.5}\rangle$.

of the values ϵ_c and critical exponents are found for the typical systems (Figure 9).

$$\frac{\partial U(l,\ \epsilon_c)}{\partial \epsilon} \sim l^{1/\nu} \tag{4}$$

$$\langle |d(l,\ \epsilon_c)|^2\rangle \propto l^{-2\beta/\nu} \tag{5}$$

$$\langle \chi(l,\ \epsilon_c)\rangle \propto l^{\gamma/\nu-2} \tag{6}$$

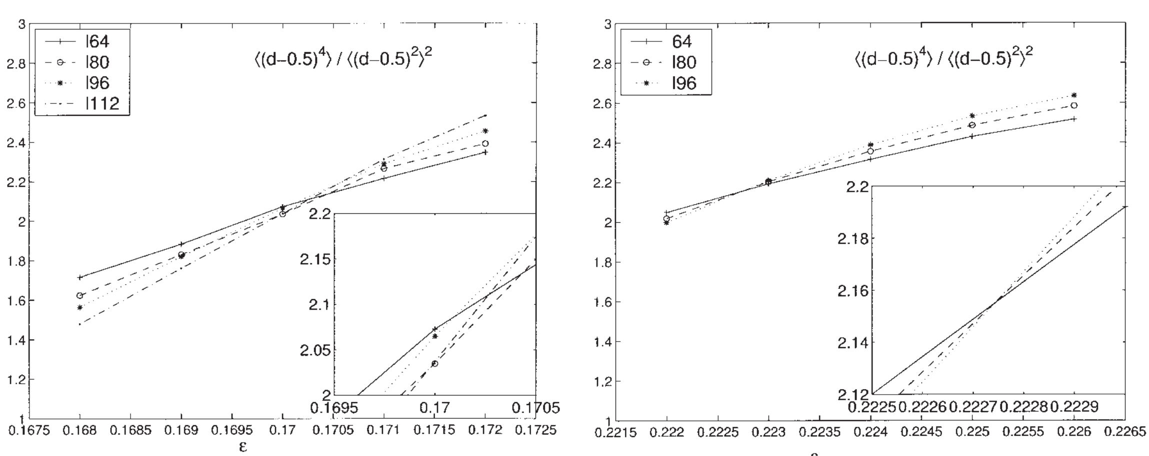
$$\langle \chi(l,\ \epsilon_c)\rangle = l^2(\langle d(l,\ \epsilon_c)^2\rangle - \langle d(l,\ \epsilon_c)\rangle^2) \tag{7}$$

4.2. Cluster Overlapping

The perfect overlap by random activation of a given number of sites is very unlikely, so $\langle o\rangle$ is useful as a supportiveness or stability measure of clusters. Table 4 shows the relationship between average activation density and the four arbitrary systems in terms of $\langle o\rangle$. The overlap at each step is normalized with the average density of that step, because an overlaps bigger than the total number of active sites cannot occur.

Having the average density level same, the overlapping is lower in less structured systems. The more supportive structure, which preserves once formed clusters, is more likely.

FIGURE 9



Estimate of ϵ_c with Binder's method for 25%(1) and 100%(4): 0.1702, and 0.2227, respectively. Estimate of critical exponents: $\nu_{25\%(1)} = 0.9504$, $\nu_{100\%(4)} = 0.9026$; $\beta_{25\%(1)} = 0.3071$, $\beta_{100\%(4)} = 0.4234$; $\gamma_{25\%(1)} = 1.1920$, $\gamma_{100\%(4)} = 0.9371$.

TABLE 4

$\langle o \rangle$ of Four Arbitrary Systems with Lattice Sizes 128×128

	$\langle \lvert d \rvert \rangle \approx 5\%$	$\langle \lvert d \rvert \rangle \approx 26\%$	$\langle \lvert d \rvert_{0.5} \rangle \approx 11\%$
Local	6.70%	52.21%	66.56%
25% (1)	6.56%	46.26%	60.66
100% (1)	6.35%	40.04%	56.44%
100% (4)	6.32%	38.00%	54.27%

Interestingly, Figure 10 shows that $\langle o \rangle$ precedes the sudden jump of $0.5 - \langle \lvert d \rvert_{0.5} \rangle$.

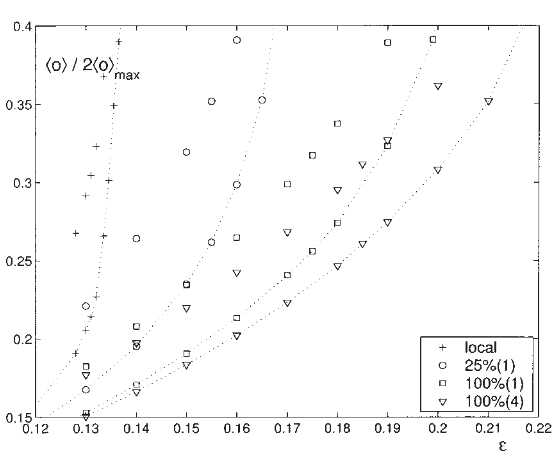
4.3. Clusters for a Jump

Starting with a lattice system with randomly initiated site activations and small value of ϵ, after a short time, the activity of the network stabilizes in either mostly active or mostly inactive mode of behavior. Because the systems are of finite size, they jump after a while from one mode of behavior to the other (Figure 11). Jumps from the mostly inactive system to mostly active, and vice versa, take a relatively short time and occur more frequently, as the ϵ increases.

What does happen with the clusters to cause the system activation jumps? In short, supportive clusters help their sites retain the same activation (Figure 7). As the clusters grow, due to the support the sites of the same activation give to each other, the jumps are more likely. The measure of proportion of clusters larger than four, $\langle c_4 \rangle$, is used to show that the proportion of supportive clusters increases before the jump, and before the system's average activation starts to change significantly (Figure 13). There are few configurations with four sites and just one is supportive, but still there have to be at least four sites in the supportive cluster. For examples, see Figure 12. Cluster configuration in higher dimensions is an unsolved problem [20]. At the present, the computer simulation techniques are more helpful than analytical approaches.

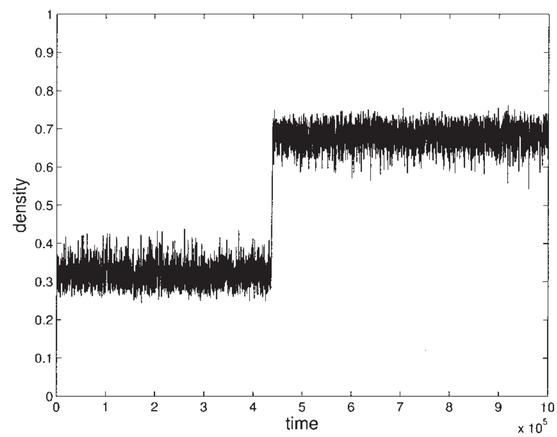
For $\langle c_4 \rangle$ as a function of ϵ, it is assumed that the similar spatial phenomenon occurs in both cases, when the jump occurs because of the system's finite size or when the jump occurs because of the sudden state transition. Because the cluster configurations are not currently solved analytically, computer simulations are used to obtain the solutions of problems with higher than one-dimension lattices.

FIGURE 10



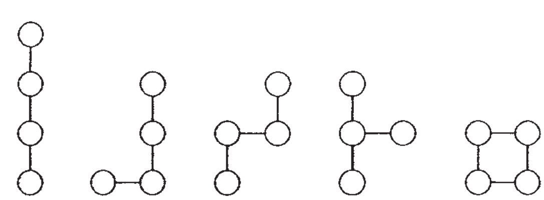
$\langle o \rangle$ vs. $\langle 0.5 - \lvert d \rvert_{0.5} \rangle$ as the function of ϵ for four arbitrary systems. Dotted lines represent $\langle 0.5 - \lvert d \rvert_{0.5} \rangle$ of *local*, 25%(1), 100%(1), and 100%(4) systems (from left). Unconnected shapes represent same systems but in terms of normalized overlaps. $\langle o \rangle$ is normalized, so it is comparable with $\langle 0.5 - \lvert d \rvert_{0.5} \rangle$.

FIGURE 11



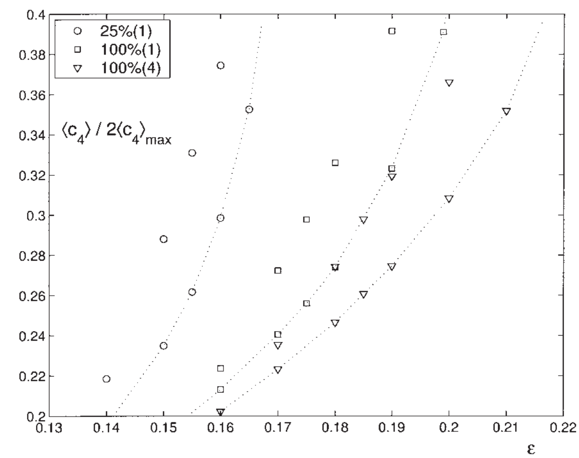
Typical temporal behavior of the active sites for 10^6 steps. Five percent of randomly selected sites in a lattice have one remote neighbor. Lattice size is 128×128. Jumps from the mostly inactive system to mostly active, and vice versa, take a relatively short time. What does happen with the clusters, to cause the system activation jumps? The size of supportive cluster helps.

FIGURE 12



All cluster configurations on the square lattice for $c = 4$. Mirror images and rotated configurations are not shown. The total number of each configuration is below the cluster image. Cluster configuration in two dimensions is an unsolved problem.

FIGURE 13



The proportion of clusters of sizes four or greater in all cluster sizes as a function of ϵ precedes the sudden jump of $\langle 0.5 - |d|_{0.5}\rangle$ (connected points). l = 128. $\langle c_4\rangle$ is normalized, so it is comparable with $\langle 0.5 - |d|_{0.5}\rangle$. Bigger and more supportive clusters prepare the system for the change of general behavior.

5. CONCLUDING REMARKS

Activation clustering behavior is described in terms of $\langle c\rangle$, $\langle o\rangle$, and $\langle c_4\rangle$. $\langle c\rangle$, $\langle o\rangle$, and $\langle c_4\rangle$ are studied as the functions of number of remote connections and as the functions of ϵ. All functions show power law behavior. In particular, $\langle c\rangle$ and $\langle |d|_{0.5}\rangle$ as the functions of ϵ, exhibit sudden transitions at the same ϵ value, while $\langle o\rangle$ and $\langle c_4\rangle$, as the functions of ϵ, precede $\langle c\rangle$ and $\langle |d|_{0.5}\rangle$ with the sudden transitions. With the addition of one way connections, the distribution of communication means is effectively increased. There are fewer repetitive links and the messages get across in fewer steps. On the other hand, with fewer remote connections, cluster structures are more static and more prone to the global change. Clustering depends on the supportive structure of the sites' connections. The more structured is the system, the more static or supportive the clusters are. Supportiveness is quantitatively described with $\langle o\rangle$. When different systems are compared, $\langle o\rangle$ values are higher in the more structured ones. What does it mean for the social and neural networks? It means that societies in which individuals mutually listen each other stronger change easier due to noisy influences. The network states in which individuals communicate in closed cycles are more fragile then the states of social networks where individuals communicate in one-way directions. $\langle c_4\rangle$ as a function of ϵ explains what occurs spatially with the clusters as ϵ is varied. It shows that the clusters get bigger and more supportive before the sudden transition is about to take place. This makes the system ready to jump to another mode of behavior. $\langle c_4\rangle$ can indicate what kind of activation clustering is necessary for a fashion fad to start, a social revolution to occur, a financial bubble to burst, or another form of state transition to emerge. With the fast cooperation of many neurons forming a brain state, the activity of a brain jumps through a trajectory of its states. To make the jump possible, supportive structures must organize to cause transitions between the thoughts, which govern the goal-directed behaviors. To understand the behaviors of neural and human societies, the mysteries of supportive structures in networks have to be explored.

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